

ON THE COEFFICIENT OF EXPANSION OF FUSED QUARTZ

BY

HOWARD D. MINCHIN

A THESIS SUBMITTED TO THE FACULTY OF THE DEPARTMENT OF
LITERATURE, SCIENCE, AND THE ARTS OF THE UNIVERSITY
OF MICHIGAN FOR THE DEGREE OF DOCTOR
OF PHILOSOPHY

1906

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THE increasing use of fused quartz in physical and chemical work and especially in optical problems renders the accurate knowledge of its physical properties very desirable. Its extremely small coefficient of expansion adapts it to investigations in which sudden changes of temperature are unavoidable and even greatly to be desired, while its transparency and high fusing point render it peculiarly suitable for the optical treatment of all problems involving wide ranges of temperature. The study of the thermal expansion of fused quartz has formed the subject of numerous recent investigations among which may be mentioned those of Le Chatelier,¹ Callendar,² and Holborn and Henning.³

Le Chatelier used a differential method, having two prisms of the same length, one of fused quartz and the other of porcelain. The latter was used as the standard, its coefficient of expansion having been determined by direct measurement. The two prisms were placed in a crucible of plumbago set in a bed of carbon which was heated to incandescence, and the expansions were measured by a cathetometer reading to 0.01 mm. The temperature was determined by means of an air thermometer. The mean coefficient of expansion of the fused quartz was found to be 70×10^{-8} between

¹ Comp. Rendus, 130, p. 1703, 1900.

² Chem. News, 83, p. 151, 1901.

³ Ann. der Physik, 10, p. 446, 1903.

the temperatures of 0° and 1000° C. A contraction was reported as taking place at about 800° C.

Callendar used a rod of fused quartz about 40 cm. long, clamped at one end, and measured the expansion by means of a micrometer microscope reading to 0.001 mm., focussed upon the free end. The rod was heated by placing it in a platinum tube carrying an electric current. The expansion of the platinum tube was determined and the temperature to which the quartz was heated was computed from the coefficient of expansion of platinum. The mean coefficient of expansion of the fused quartz between room temperature and 1000° C. was found to be 59×10^{-8} . The expansion was found to be regular up to 1000° but from 1000° to 1400° C. it increased rapidly, and beyond 1400° a contraction took place.

Holborn and Henning used a rod of the material about 52 cm. long and observed the change in length by means of telescopes focussed upon marks cut near the ends of the rod. The rod was heated electrically¹ by placing it in a porcelain tube 80 cm. long mounted horizontally. Temperature measurements were made by means of a thermoelement, and the temperature of the rod was determined at nine different points throughout its length. Observations were made at room temperature, at 250° , 500° , 750° , and 1000° C. As a mean of fourteen determinations 54×10^{-8} is given as the coefficient of expansion.

The marked discrepancy in the results thus far obtained seemed to justify a redetermination of this important constant, with more delicate apparatus for measuring the expansion and especial devices for maintaining the temperature of the expanding solid both uniform and constant.

METHOD.

The method used in this investigation for measuring the coefficient of expansion is briefly as follows: A ring of the fused quartz about 1 cm. long is placed in an electrically heated oven and the expansion is determined by the displacement of interference fringes. The higher temperatures are determined by means of a thermoelement and the lower temperatures by means of mercury thermometers. The method of determining the expansion by the dis-

¹ Ann. der Physik, 2, p. 505, 1900.

placement of interference fringes is a modification of the method first employed by Fizeau.

The fused quartz system (Fig. 1) is arranged for the measurement of the expansion in the manner first devised by Pulfrich¹ and used by Reimerdes² and later by Randall.³ The bed-plate *A* is of fused quartz 38.4 mm. in diameter and 10 mm. thick, and with its two faces ground approximately parallel. Its upper surface is plane polished, while the lower surface is left rough and ground in such a way as to leave three equidistant feet so that when the plate is mounted upon a level surface its upper surface is horizontal. The piece of fused quartz *Q*, whose coefficient of expansion was to be studied, is in the form of a ring approximately 9.9 mm. long, 31.8 mm. in external diameter, and 24.4 mm. internal diameter. This ring was made from a circular plate

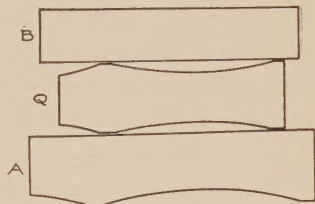


Fig. 1.

whose two surfaces have been ground and polished accurately plane parallel. From the center of this plate was cut a cylinder 24.4 mm. in diameter. Each end has been ground away so as to leave three equidistant feet as shown in the figure. The cover-plate *B*, also of fused quartz, is 7.5 mm. thick and 36.0 mm. in diameter. Its surfaces are plane polished and inclined to each other at an angle of about 20 minutes in order to avoid the disturbing effect due to the light reflected at the upper surface. A small plate *C*, not shown in the figure, made from the same material as the other pieces is approximately 19.8 mm. in diameter and about 0.2 mm. thick. Its upper surface is plane polished and its lower surface is rough and ground away so as to leave three feet similar to the arrangement of the bed-plate.

In arranging the parts preparatory to making a series of observations the plate *C* is placed upon the bed-plate *A* and the ring *Q* is then placed upon *A* symmetrically about *C*. The cover-plate *B* is next placed upon the ring *Q*. One of the feet of *C* was ground

¹ Zeitschr. für Krystall., 31, p. 372, 1899.

² Inaug. Diss., Jena, 1896.

³ PHYS. REV., 20, p. 10, 1905.

off sufficiently to make the angle included between C and the lower surface of B of the proper size to give interference fringes of the desired width. For the purpose of measurement the cover-plate B has at the center of its lower surface a small fine circle and around this a number of concentric circles. The inner circle serves as a fixed point in the field of view from which measurements are made. Upon the upper surface of B is a fine mark cut parallel to the thin edge of the plate. This mark is to enable the operator to place the parts in the same relative position, each time the system is set up.

The fused quartz parts were made from material produced in the smelting works of the firm of Carl Zeiss, Jena, and the ring and optical surfaces were also made by Zeiss.

When the apparatus is clean and properly adjusted and illuminated by monochromatic light the eye will see the surface of the plate C crossed by a system of parallel interference fringes. When the illumination is produced by light of different colors a system of rainbow colored fringes can be observed. Each dark band of the fringe system represents a definite thickness of the air layer between B and C . Let λ be the wave-length of the light used, then the thickness of the air layer under any dark band differs from the thickness under the next dark band on either side by $\frac{1}{2}\lambda$. When the fused quartz system is heated the ring Q expands and the plates B and C are separated and the interference fringes move toward the thin edge of the included air layer. The number of dark bands passing under the small reference circle on B represents the change in the thickness of the air layer immediately under this circle and thus measures the expansion of the ring Q in terms of the wave-length of the light employed. The total expansion for any temperature interval $(t_2 - t_1)$ is $f \times \lambda/2$, where f is the number of dark bands passing under the reference circle for a temperature change from t_1 to t_2 , and λ is the wave-length of light used. We then have the coefficient of expansion α , given by the equation

$$\alpha = \frac{f \frac{\lambda}{2}}{L(t_2 - t_1)},$$

where L is the effective length of the fused quartz expanding, that

is, the length of the ring Q , minus the length of the small plate C .

The spectral lines used for measurements were the mercury green ($\lambda = 546 \mu\mu$) and the yellow lines of mercury ($\lambda = 578.8 \mu\mu$ and $\lambda = 576.8 \mu\mu$). The hydrogen red ($\lambda = 656.2 \mu\mu$) was also used for some measurements but the intensity of illumination was not satisfactory and its use was abandoned.

APPARATUS.

The interferometer employed was an Abbé-Fizeau instrument as modified and improved by Pulfrich, and constructed by Carl Zeiss, Jena. The same instrument was used by Randall,¹ who has given a full description with detailed directions for its manipulation in connection with his investigation upon crystalline quartz. As a source of light the ordinary H-form of Geissler tube containing hydrogen and vapor of mercury was used.

THE SUPPORTING SYSTEM.

To avoid the presence of metal in the furnace the support for the quartz system was constructed as follows: Two rods FF (Fig. 2), of Berlin porcelain, about 50 cm. long, are clamped to a metal block E , by means of metal straps and held in position by screws. These rods carry at their lower ends a porcelain plate M . The block E , is made tapering in order to admit the interferometer tube between FF , and also as an aid in holding M in place. The rods FF , are notched at their lower ends to fit into slots cut in M as shown in the figure. This holds M sufficiently rigid to furnish a stable support for the optical system.

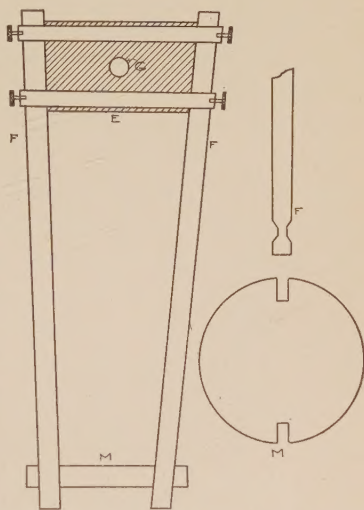


Fig. 2.

To mount the support a threaded hole G (Fig. 2), is made in the center of E , into which is screwed a thirteen millimeter steel rod.

¹ Loc. cit.

This rod is held by a right angled clamp to a larger rod attached rigidly to the pier. This method of mounting gives to the support two adjustments for levelling. It may be revolved on the rod attached to *E* in a plane at right angles to the axis of the interferometer tube, and also around the second rod in the plane of the axis of the interferometer tube. This gives sufficient freedom of movement to permit of accurate adjustment of the interference system.

THE ELECTRIC FURNACE.

The furnace consists of an unglazed porcelain battery jar 14.6 cm. inside length and 6.9 cm. inside diameter. A thin sheet of chemically pure asbestos paper was wound around this cylinder and then upon this two coils of rolled platinum wire were wound with the strands about 2 mm. apart. Fourteen feet of number 20 platinum wire was used. This was rolled so that its width was about twice its original diameter when its length was about 28 feet. The coils were wound separately so that either coil may be used alone, both coils may be used in parallel, or the two coils may be used in series. The plan of winding and the connections are shown in cross-section in Fig. 3. The resistance of each coil at room temperature is about 2.5 ohms.

The coils were covered with about a half inch coating of ganister made into a mortar. This material is a mixture of six parts of white sand and one part of clay ground together very fine. This was then covered by several layers of asbestos paper with small strips of asbestos between so as to keep the layers separated from each other for the purpose of lessening the radiation. The asbestos winding was carried about two inches above the porcelain cylinder. The whole was then packed in mineral wool, in a sheet-iron cylinder

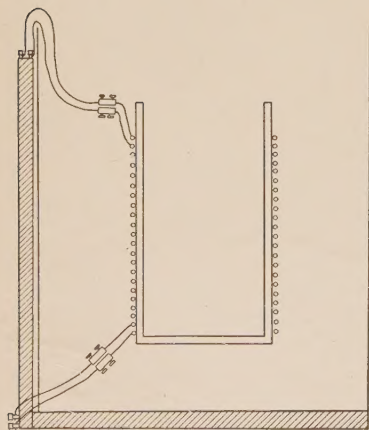


Fig. 3.

and mounted upon a table capable of being raised or lowered by

means of a screw actuated by a crank. This table was sufficiently rigid to prevent any lateral displacement of the oven, or jarring of the optical parts.

The cover was a porcelain plate similar to M , and split through the middle so that it could be put in place when the supporting system was adjusted in the furnace. Openings were made in it for the thermometers and thermoelements and also to admit the light. The arrangement of the cover is shown in Fig. 4, where A is the central opening about one inch in diameter to admit the light, bb are openings for the thermometers or thermoelements, and cc are the openings for the supporting rods FF .

In order to prevent radiation through A and yet not cut off the light a series of air chambers was built up. The opening A was

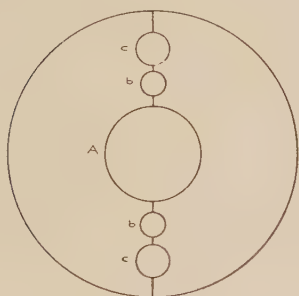


Fig. 4.

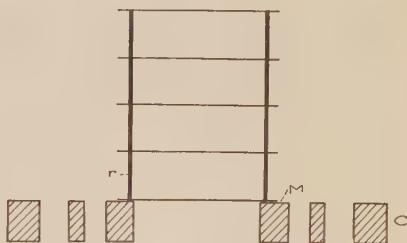


Fig. 5.

covered with a thin sheet of mica, then a collar or ring of glass, cut from a glass tube about one inch in diameter, was placed on the mica and over the opening, this ring was covered by a second sheet of mica, another glass collar added and another covering of mica. In this way four air chambers were built up, which served as an excellent check to radiation and did not noticeably diminish the illumination of the fringe system. A cross-section of the arrangement of the air chambers is shown in Fig. 5, where C is the cover, M the mica sheets and r the glass collars.

The effectiveness of the heat insulation was shown by the fact that the sheet-iron cylinder in which the furnace was packed showed but little rise in temperature when the furnace was heated up to 1000° C. Two thermometers were hung above the furnace, one close to the supports FF and within a few centimeters of the inte-

rior of the furnace, near the air chambers above *A*, the other just outside of the asbestos packing, and these showed temperatures but a few degrees above that of the room in each case. The small mass of the furnace renders it possible to heat up very quickly and at the same time allows the temperature of the interior to follow very closely every change in the heating current. The temperature can be held constant, to the full limit of the accuracy of the temperature measurements, for hours at a time. A current of 5 amperes is sufficient to heat up to 1000° C. in about two hours. In order to test the uniformity of the temperature within the heated space a series of temperature determinations was made

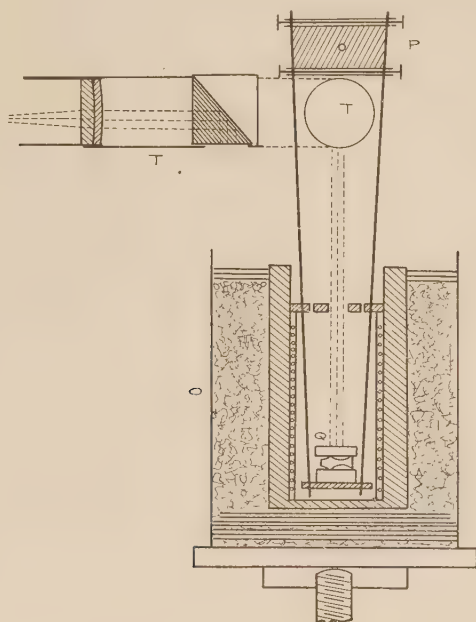


Fig. 6.

in the different regions of the furnace. Two thermoelements were used and so adjusted that the temperatures of two separate regions of the furnace were determined at the same time. The temperature was found to be constant to within $0^{\circ}.5$ throughout the whole interior, no variation being detected until the one element was brought within a centimeter of the top of the furnace and this variation disappeared when the cover and insulating material had been properly adjusted.

The apparatus was set up in a basement room of the Physical Laboratory of the University of Michigan. A brick pier served as a support for the interferometer and the supporting system. The electric oven and other devices were mounted upon the asphalt floor. In Fig. 6 is shown a vertical section of the end of the interferometer tube *T*, the electric oven *O*, the supporting system *P* and the quartz system *Q* when adjusted for work.

During the first part of the investigation the direct current, at 220 volts pressure, from the university lighting plant was used. Later it became necessary to use current from the storage battery for heating. This battery gave but 70 volts, yet owing to the low resistance of the coils of the furnace sufficient current was available for heating up to 1000° C. or more, while the extreme steadiness of the current proved to be a great advantage when it was necessary to keep the temperature constant at any point above the temperature of the room.

Table I. gives an idea of the performance of the furnace and its condition as regards uniformity and efficiency for the various temperature intervals worked over.

TABLE I.

Temperature Interval Worked Over.	Mean Current Used to Heat Up.	Time Required to Heat Up.	Current Required to Hold Temperature Constant.	Time During Which the Temperature was Held Constant.
	Amperes.	Hours. Min.	Amperes.	Hours. Min.
Room temperature to 300°	3.0	1 05	2.4 to 2.5	2 20
" " " 400	3.0	1 10	3.2	2 30
" " " 500	3.5	1 20	3.3 to 3.4	2 50
" " " 600	5.0	1 16	3.6 to 3.8	2 00
" " " 700	5.0	1 30	3.9	2 45
" " " 800	5.0	1 50	4.6 to 4.8	2 20
" " " 900	5.0	2 00	4.5 to 4.6	2 00

In calibrating the thermoelements mentioned above, the furnace was heated up to 1350° C. and it was later heated up to 1200° C. on a trial test for the time required to heat up. For this test 5.5 amperes were used and the time required to reach 1200° C. was two hours and ten minutes.

THERMOMETERS.

Four thermometers were used. For room temperatures a Peters and Rost thermometer reading to 60° C. and calibrated in the physical laboratory was used. For temperatures up to 200° C. two standard thermometers, made by Haak of Jena, were used, the one graduated from -20° C. to 105° C., the other from 90° C. to 210° C. These were calibrated at the Reichsanstalt. The fourth thermometer is graduated in degrees, having a short scale at 0° C. and after an enlargement of the bore the graduation extends from 195° C. to 550° C. This thermometer was also calibrated at the Reichsanstalt. For temperatures between 300° C. and 1000° C. a platinum-platinum-rhodium thermoelement was used. This element was certified by the Reichsanstalt. As a check on the latter a platinum-platinum-iridium thermoelement, made by myself and calibrated in the physical laboratory, was used. As a further check both these thermoelements were compared with a platinum-platinum-rhodium element, certified by the Reichsanstalt, and belonging to Professor E. D. Campbell, director of the chemical laboratory.

In measuring the temperature by means of the thermoelement the method used was the compensation method described by Lindeck and Rothe.¹ The apparatus was made by Siemens and Halske according to specifications of the Reichsanstalt.

THE SPHEROMETER.

The spherometer used in measuring the fused quartz ring and plates was one made by the Geneva Society and reads to 0.001 mm. The dimensions of the different parts of the fused quartz used in the computations were obtained by taking the mean of several series of readings. The length of Q is the mean of thirty readings, and the length of C is the mean of twenty-four readings.

ADJUSTMENT OF THE APPARATUS.

The adjustment of the interferometer and the Geissler tube was made according to the method described by Randall.² The support

¹ *Zeitschr. für Instr.*, 20, p. 285, 1900.

² *Loc. cit.*

P (Figs. 2 and 6) was mounted independently upon rods clamped to the pier and so arranged that it could be readily adjusted for the proper level. The thermoelement K (Fig. 7), was fastened to one of the supporting rods F , so that it passed through one of the openings of the cover of the furnace. Its junction J , was placed within about two millimeters of the ring Q , and between A and B as indicated in the figure.

In the adjustment of the system great care was necessary to avoid the presence of dust or lint between the surfaces at points where contact was desired, therefore each piece was thoroughly cleaned before being set in place. Plate A was placed centrally upon M of the support, C was placed in the center of A , the ring Q was then placed symmetrically around C , and the plate B placed on Q with the circles down. The support was then levelled and the optical system adjusted until interference fringes were obtained. The different pieces were marked to aid in placing them in the same relative position at each mounting of the system. When the

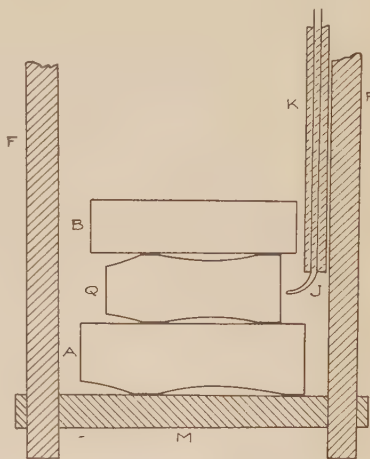


Fig. 7.

fringes were erect and sufficiently bright readings were made upon two adjacent dark bands to determine the width of the fringes. To insure against the presence of dust or foreign particles between the surfaces in contact, the fused quartz system was taken down and remounted several times, the width of the fringes being determined each time. When several successive determinations gave the same band width it was assumed that the apparatus was in proper adjustment for work. The reference circle was then adjusted so that it was in the center of the field in order that any rectification of the fringes should still leave them symmetrically placed in the field.

The furnace was then raised until the fused quartz system was near the bottom and symmetrically placed in the interior, the cover

C, was then put in place, the air chambers built over the opening *A*, and the space around these chambers packed with asbestos up to the asbestos winding on the furnace. For the lower temperatures a thermometer was introduced in the second opening *b*, and for the higher temperatures the second thermoelement was used in a manner similar to the one already described. The apparatus was allowed to stand over night in order to allow it to acquire a stationary temperature. The position of the fringes was read and the temperature and barometric pressure determined. The current was then turned on and the system heated up to the desired temperature. The current was then adjusted so as to maintain this temperature constant and micrometer readings were made upon some one of the dark bands in the field of view. The temperature was kept constant and at intervals of about 20 minutes the position of the dark band selected was again determined. After the temperature had been held constant for about an hour the position of the band did not change. This indicated that the system had acquired a stationary temperature. The final readings on the bands were taken after the temperature had been held constant for a period of not less than two hours. In all micrometer readings made upon the fringes a mean of ten or more readings was taken. During the heating of the quartz the number of dark bands passing the reference circle were counted. When the readings at the desired temperature had been made the current was shut off and the system was allowed to cool to room temperature and another set of readings made. Thus for each temperature interval worked over, two separate determinations were made, one from the lower temperature to the higher and a second from the higher to the lower temperature. In this way separate determinations were made over the temperature intervals between room temperature and each of the temperatures 200°, 300°, 400°, 500°, 600°, 700°, 800°, 900° and 950° C. After each of the series of readings 1, 2, 5 and 6 given in Table II., the quartz was removed from the furnace, taken down, remounted and the furnace again adjusted for the next series of readings. The quartz pieces were all carefully measured after the heating was completed and their lengths were found to be the same as at the beginning.

Two methods were used for determining the number of fringes displaced, the one by direct observation, the other by computation.

By means of the micrometer attached to the eye-piece of the interferometer the exact position of the bands was determined and then the number of whole bands passing the reference circle was counted while the system was being heated up to the desired temperature. To this whole number was added the fraction of a band by which the last dark band had passed the reference circle, and from this was subtracted the fraction of a band by which the first dark band was past when the counting began. The total fringe displacement was then given by the formula

$$f = M + \partial_{t_2} - \partial_{t_1},$$

where M represents the number of whole bands observed and ∂_{t_2} and ∂_{t_1} the fractions of a band at temperatures t_2 and t_1 . These quantities were determined from the micrometer readings. The heating and the expansion were so regular that once having determined the time of passage of a dark band it was an easy matter to check up the result.

The method by computation is that first described by Abbé.¹

As a change in the temperature and pressure results in a change in the index of refraction of the air layer between the reflecting surfaces, a correction was made upon the observed number of fringes displaced. The correction was computed by the following formula given by Pulfrich:²

$$K = d(t_2 - t_1) \frac{b_1}{760} \cdot \frac{1}{1 + at_1} \cdot \frac{1}{1 + at_2} \left[2 \frac{(N - 1)a}{\lambda} \right] \\ + d(b_2 - b_1) \frac{1}{1 + at_2} \left[2 \frac{N - 1}{\lambda} \cdot \frac{1}{760} \right]$$

in which K represents the correction, d the thickness of the air

¹ Wied. Ann., 38, p. 473, 1889.

² Zeitschr. für Instr., 13, p. 455, 1893. While the relation here assumed to exist between the index of refraction of a gas and its temperature and pressure, is known to hold for temperatures up to 100° C., no data, so far as is known to the writer, exist for the behavior of a gas at such temperatures as are reached in this work. An investigation of the validity of this correction for temperatures between 100° and 900° C., is being made by Dr. Randall.

layer = $Q - C$ (Fig. 1), b_1 the barometric pressure observed at the lower temperature t_1 , b_2 the barometric pressure at the higher temperature t_2 , α the coefficient of expansion of air, N the index of refraction of air under standard conditions and λ the wave length of light used.

THE DEGREE OF ACCURACY.

The formula for the coefficient of expansion

$$\alpha = \frac{f \frac{\lambda}{2}}{L(t_2 - t_1)}$$

shows that α is dependent upon the variables f , L , t_2 , and t_1 . In the expression $f = M + \partial_{t_2} - \partial_{t_1}$ the ∂ 's could be determined with an error affecting f by ± 0.02 of a fringe and the resulting error in the value of α for the larger temperature intervals is $\pm 0.1 \times 10^{-8}$ and for the smaller temperature intervals it is $\pm 0.5 \times 10^{-8}$. The length of L could be determined to within ± 0.001 mm. and the error in α due to this error is 0.01×10^{-8} . The temperature measurements made with the thermometers could be determined with an error of $\pm 0.1^\circ$ C. With the thermoelement the temperature could be determined with an error of $\pm 0.5^\circ$ at the lower temperatures and $\pm 2^\circ$ at the higher temperatures. The error in α resulting from the measurement of the temperature intervals is $\pm 0.1 \times 10^{-8}$ and $\pm 1.1 \times 10^{-8}$ respectively. The total effect of these errors upon the final result is given by the equation

$$E = \sqrt{E_f^2 + E_L^2 + E_t^2}$$

where E_f , E_L , and E_t represent the error in the measurement of the number of bands, the length of the ring, and the temperature interval, respectively. For the shorter temperature intervals at the higher temperatures, this amounts to 1.6×10^{-8} and for the longer intervals it is 0.5×10^{-8} .

The series of readings in Table II. were made for the temperature intervals indicated.

After the above series of readings were made the apparatus was dismantled and the quartz system, the furnace, and the thermoele-

TABLE II.

Date.	Series.	Temperature Interval Worked Over.	
		(1)	(2)
1905, Sept. 2	1	Room temperature to 200°	Back to room temperature.
" 7		" " " 300	" " " "
" 8		" " " 400	" " " "
" 9		" " " 500	" " " "
" 10		" " " 600	" " " "
" 11		" " " 700	" " " "
" 12		" " " 800	" " " "
" 13		" " " 900	" " " "
" 13		" " " 950	" " " "
" 14		" " " 600	" " " "
" 14		" " " 800	" " " "
" 15		" " " 300	" " " "
" 15		300° " 400	
" 15	2	400 " 500	
" 15		500 " 600	
" 15		600 " 700	
" 16		Room temperature to 800°	Back to room temperature.
" 16	3	800° " 900	
" 16		900 " 950	

ments were taken to the University of Rochester where the apparatus was again set up. A new Abbé-Fizeau interferometer belonging to the University of Rochester was used, and for measuring the temperature an Otto Wolff potentiometer was used in connection with the thermoelements in place of the compensation device of Siemens and Halske. The series of readings given in Table III. were then made.

TABLE III.

Date.	Series.	Temperature Interval Worked Over.	
		(1)	(2)
1905.	6	Room temperature to 300°	Back to room temperature.
Nov. 8		" " " 500	" " " "
" 10		" " " 700	" " " "
" 11		" " " 900	" " " "
" 22		" " " 200	
Dec. 5	7	200° to 400	
" 18		400 to 600	
" 19		600 to 700	
1906.	8	700 to 800	
Jan. 16			
" 18			

EXAMPLE OF A COMPLETE DETERMINATION.

September 7, 1905.

Length of fused quartz ring, 9.9009 mm. Effective length of fused quartz
 Length of fused quartz cyl- expanding, 7.9513 mm.
 inder, 1.9496 mm. Thickness of air layer, 7.9513 mm.

Computation for one set of readings:

$$t_1 = 18.4 \text{ C.}, \quad b_1 = 742.0 \text{ mm.}$$

$$t_2 = 300^\circ \text{ C.}, \quad b_2 = 742.6 \text{ mm.}$$

$$\delta_{t_1} = 0.2736, \quad \delta_{t_2} = 0.3981 \text{ mm.}$$

$$M \text{ observed} = 3, \quad \lambda = 0.000546 \text{ mm.}$$

$$f = 3 + 0.398 - 0.274 = 3.124.$$

$$K = 0.385.$$

$$f_0 = 3.124 + 0.385 = 3.509.$$

$$f_0 = \text{number of bands when correction is applied.}$$

$$a = \frac{f\lambda/2}{L(t_2 - t_1)} = 43 \times 10^{-8}.$$

RESULTS.

Table IV. contains the data obtained in the physical laboratory of the University of Michigan. Table V. contains the data obtained in the physical laboratory of the University of Rochester.

TABLE IV.

Series.	Temperature Interval Worked Over.				
	Lower to Higher.		a	Higher to Lower.	
1	Room temperature to 200°		43.2×10^{-8}	Back to room temperature.	43.3×10^{-8}
	"	" 300	43.0	" " " "	43.2
	"	" 400	44.0	" " " "	44.5
	"	" 500	46.0	" " " "	45.7
	"	" 600	44.0	" " " "	44.8
	"	" 700	44.0	" " " "	44.1
	"	" 800	46.0	" " " "	45.8
	"	" 900	43.0	" " " "	44.0
	"	" 950	44.0	" " " "	44.2
2	"	" 600	46.0	" " " "	45.7
3	"	" 800	46.0	" " " "	45.8
4	"	" 300	44.0		
	300°	" 400	44.0		
	400	" 500	46.0		
	500	" 600	45.0		
	600	" 700	45.0		
5	Room temperature " 800		46.0		
	800°	" 900	44.0		
	900	" 950	45.0		

TABLE V.

Series.	Temperature Interval Worked Over.				
	Lower to Higher.		α	Higher to Lower.	
6	Room temperature to 200°		44.0×10^{-8}	Back to room temperature.	
	"	" 500	46.0	" " " "	44.5×10^{-8}
	"	" 700	44.8	" " " "	45.4
	"	" 900	45.0	" " " "	45.2
	"	" 200	43.0	" " " "	45.1
7	200°	" 400	45.0		
8	400	" 600	44.5		
	600	" 700	45.0		
	700	" 800	46.0		

CONDITIONS FOR OBSERVATIONS.

At about 400° C. the interior of the furnace began to appear bright and at about 500° C. it gave a reddish tint. This light, however, did not interfere with the readings upon the interference fringes. Between 700° C. and 800° C. a band of reddish light emitted by the fused quartz appeared in the telescope of the interferometer in the neighborhood of the hydrogen red of the Geissler tube. At about 825° C. this light became so bright that it annulled the hydrogen red in the field, and above 950° C. the intensity of the light from the fused quartz was so great that further readings were impossible. The colors used in the interference phenomenon were the yellow and the green lines of mercury, and up to 950° C. the brilliancy and sharpness of the fringes due to these lines were as marked as at the lower temperatures. Even at this high temperature there was no wandering or quivering of the fringes, and readings could be taken as readily as at the lower temperatures. This was due to the freedom from air currents and to the perfect insulation of the furnace. In so far as the writer is aware interference fringes have not been previously used for measurements at temperatures above 500° C.

In the construction of the interferometer the light is analyzed by a direct vision prism train in the eye-piece, and it is possible that some modification of this set of prisms might render it possible to extend the temperature limit at which the fringes could be read to 1100° C. or 1200° C. The use of some element in the Geissler tube which would give brilliant lines in the blue or violet would certainly extend this range.

COMPARISON OF RESULTS WITH THOSE OF OTHER OBSERVERS.

The results obtained in the investigations referred to in the introduction are given below together with that obtained by the writer.

	α
Le Chatelier	50×10^{-8} ,
Callendar	59×10^{-8} ,
Holborn and Henning	54×10^{-8} ,
Minchin	45×10^{-8} .

It is seen that the result obtained by the writer is considerably below the other results. Le Chatelier in his work used the differential method, using porcelain as his standard. He found certain irregularities in the expansion of the fused quartz, but Callendar thinks it probable that this was due to the irregularities inherent in the expansion of porcelain. Shenstone¹ says the fused quartz used by Le Chatelier was probably impure, and Le Chatelier himself expresses doubt as to its purity.

Callendar placed a rod of the fused quartz in a platinum tube and clamped both at one end and read the expansion by means of a micrometer microscope reading to 0.001 mm. His method of determining temperatures was, to say the least, indirect and liable to error.

The value $\alpha = 54 \times 10^{-8}$, given by Messrs. Holborn and Henning as the result of their measurements, was a mean value obtained from several different ranges of temperature. The following table contains, in the first two columns the data given by Holborn and Henning,² to which has been added in the third column the corresponding value of α computed by the writer.

The effective length of the fused quartz expanding was 486.9 mm.

As will be noted from the table, the individual values of α vary considerably, and while several of these values are comparable to the result obtained in this investigation, still the differences between individual values of the series are sufficiently large to warrant some consideration of the method employed.

The method used by Messrs. Holborn and Henning, while offering the seeming advantage of a great length of the expanding substance, was weak in that it failed to secure uniform temperature

¹ Nature, 64, p. 65, 1901.

² Ann. der Physik, 10, p. 447, 1903.

throughout the length of the rod, the temperature of the ends being ten per cent. lower than that of the middle.¹ It was also found necessary to correct for the expansion of the supports of the reading microscopes and it seems possible that the method employed might involve other errors due to expansion, which were not entirely eliminated by the foregoing correction. In discussing the rather wide variation between their individual results, they suggest that this may possibly be due to the irregular form of the fused quartz rod under examination.² By means of their apparatus Messrs. Holborn and Henning were able to measure elongations to 0.001 mm., which in a rod of 50 cm. length would amount to 1 part in 500,000.

TABLE VI.

Temperature Interval Worked Over.	Expansion for Interval in μ .	α	Temperature Interval Worked Over.	Expansion for Interval in μ .	α
17° to 250°	54.0	47.6×10^{-8}	500° to 750°	68.0	55.9×10^{-8}
	57.5	50.7 "		62.0	50.9 "
				60.0	49.3 "
	54.0	47.6 "		62.5	51.3 "
	52.0	45.8 "		192.5	53.9 "
250° to 500°	79.5	65.3 "	16° to 750°		
	74.0	60.8 "			
	77.0	63.3 "	750° to 1000°	66.0	54.2 "
				61.0	50.1 "

The method and the apparatus employed in the present investigation seem to justify confidence in the results obtained for the following important reasons.

(a) *Uniformity of Temperature Throughout the Expanding Body.* — As has been pointed out, the temperature of the interior of the electric furnace did not vary by as much as 0°.5 throughout a range of about 14 cm., and the length of the expanding quartz, being approximately 1 cm., the error from inequalities of temperature in its different parts may be considered as eliminated.

(b) *Constancy of Temperature During Measurements.* — The temperature of the furnace and its contents was held constant during the measurements or at least the variations of temperature were

¹ Ann. der Physik, 2, p. 507, 1900.

² Ibid., 10, p. 447, 1903.

smaller than could be detected by means of the apparatus employed, *i. e.*, the compensation apparatus, furnished by Siemens and Halske. The use of the current from the storage battery contributed greatly to this result.

(c) *Uniformity and Homogeneity of the Quartz Specimen Examined.* — The quartz ring employed, received from the smelting works of the "Zeiss Fabrik" may safely be assumed to be an excellent sample of pure fused quartz, while its small length and the symmetry of its parts render errors from any want of symmetry, or lack of homogeneity extremely unlikely.

(d) *Close Agreement Between Temperature Measured and Actual Temperature of the Quartz Ring.* — The junction of the thermoelement was placed within two millimeters of the surface of the quartz ring and since the variation of temperature in the interior of the furnace was practically *nil*, and since the temperature of the junction was constant both before and during the measurements, it may be assumed that the indicated temperature was very accurately that of the expanding quartz.

(e) *Variety of Temperature Ranges and of Conditions of Measurement.* — No pains were spared to make the variety of temperature intervals over which the expansion could be measured as great as possible, in order to detect any variation in the rate of expansion of the fused quartz within that limit, if such variation existed. Also the repetition of the measurements, in another laboratory, and under new conditions, and with new instruments is thought to give added value to the result obtained.

(f) *Delicacy of Measurements of Elongation.* — With the system as finally arranged for measurements, the fringe width was equivalent to 100 divisions of the divided head of the micrometer, for this rendered it possible to measure displacements to one two-hundredth of a wave-length. Hence elongations of double this amount, or 0.01 of a wave-length of the light employed, could be measured with ease. This gives, in the case of the yellow light from mercury vapor, the possibility of measuring elongations of the order of $0.000578 \times 0.01 = 0.00000578$ mm., and this when used in connection with a length of 7.9 mm. of the expanding substance, amounts to one part in 1,500,000.

SUMMARY OF RESULTS.

I. In this investigation it has been shown that interference methods are applicable to the measurement of minute lengths up to 950° C. when the fringes are produced between surfaces of fused quartz; and that up to this temperature the clearness and the brilliancy of the fringes are as great as at the lower temperatures.

II. The expansion of fused quartz is uniform from room temperature to the highest temperature observed. No change in the expansion, either as an increase or a sudden decrease, as was reported by Le Chatelier, was observed.

III. No after effects due to the heating were observed.

IV. The mean value of the coefficient of expansion of fused quartz, as deduced from 43 determinations, is

$$a = 44.9 \times 10^{-8}.$$

In conclusion the writer desires to express his gratitude to Professor Reed, under whose immediate direction this investigation was pursued, for the many suggestions offered, the kindly interest taken during the progress of the work, and for the generous loan of apparatus with which to pursue to completion the work at the University of Rochester. Thanks are also due to Dr. H. M. Randall for valuable suggestion and assistance in overcoming experimental difficulties.

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